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Higher spin fields and the problem of cosmological constant.

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Abstract

The cosmological evolution of free massless vector or tensor (but not gauge) fields minimally coupled to gravity is analyzed. It is shown that there are some unstable solutions for these fields in De Sitter background. The back reaction of the energy-momentum tensor of such solutions to the original cosmological constant exactly cancels the latter and the expansion regime changes from the exponential to the power law one. In contrast to the adjustment mechanism realized by a scalar field the gravitational coupling constant in this model is time-independent and the resulting cosmology may resemble the realistic one.

1 Introduction

The mystery of the cosmological constant Λ is one of the most profound or just the most profound one in modern fundamental physics. Astronomical observations show that it is extremely small on the scale of elementary particle physics while it still may be cosmologically essential. The astronomical bounds (see e.g. the review[1]) on the vacuum energy density $\rho_{vac} = \Lambda m_{Pl}^2/8\pi$ are roughly speaking that ρ_{vac} does not exceed the value of the critical energy density $\rho_c = 2 \cdot 10^{-29} h_{100}^2 \text{ g/cm}^3$. (Here h_{100} is the Hubble parameter in units 100 km/sec/Mpc.) Written in terms of particle physics units the bounds reads:

$$\rho_{vac} < 10^{-47} \text{GeV}^4$$
(1)

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On the other hand there are plenty contributions coming from different physical fields which by many orders of magnitude exceed the permitted value (1). For the review see papers [2, 3]. For example the energy density of the chiral condensate $\langle \bar{q}q \rangle$ well established in Quantum Chromodynamics (QCD)[4] is approximately $\rho_{qq} = 10^{-3} - 10^{-4} \text{ GeV}^4$ and the energy density of gluon condensate[5] is $\rho_{GG} = 10^{-3} - 10^{-4} \text{ GeV}^4$ which are at least by 44-45 (!) orders of magnitude larger than the astronomical bound on the vacuum energy (1). There could be some other contributions which are even much bigger than ρ_{qq} or ρ_{GG} . In particular supergravity or superstring models naturally imply $\rho_{vac} \approx m_{Pl}^4/(8\pi)^2$ which exceeds the bound (1) by approximately 120 orders of magnitude.

It is hard, or better to say, impossible to believe to an accidental cancellation with such a precision by a contribution from some other fields, which know nothing about quarks and gluons, so one is forced to find a mechanism which can somehow achieve that dynamically. This is definitely the problem of low energy physics because the characteristic scale at which this mechanism should operate is above $l = \rho_{vac}^{1/4} \approx$ $(10^{12} {\rm GeV})^{-1} \approx 10^{-2} {\rm cm}$. The natural idea is to invent an adjustment mechanism [6, 7, 9, 10, 11, 12] realized by a new massless (or extremely light) classical field with not necessarily positive definite energy density. The interaction of this field with the curvature of space-time should be chosen in such a way that its energy-momentum tensor would cancel down the underlying vacuum energy. It resembles the axionic solution of the problem of strong CP-violation in QCD [13, 14, 15]. However the attempts to realize the adjustment with a scalar field proved to be unsuccessful. The free massless scalar field obeying the equation of motion $D^2\phi = 0$ (where D is the covariant derivative in the gravitational background) is stable, or in other words it does not possess solution rising with time and its stress tensor asymptotically vanishes. However a scalar field non-minimally coupled to the curvature as $\xi R\phi^2$ is indeed unstable in De Sitter space-time (if $\xi R < 0$)[6] and its back reaction turns the exponential expansion, $a(t) \sim \exp(Ht)$ into a power law one, $a(t) \sim t^{\sigma}$ but at the expense of the asymptotically vanishing gravitational constant $G_N \sim 1/t^2$. Moreover the energy-momentum tensor of such scalar field is not proportional to the vacuum energy-momentum tensor $T_{\mu\nu}^{vac} = \rho_{vac}g_{\mu\nu}$ but has a quite different tensor structure.

It has been argued[6, 8, 3] that despite the absence of satisfactory models one still can conclude that an adjustment mechanism generically leads to a non-complete cancellation of the vacuum energy with the non-compensated remnant of the order of $\delta\rho \sim m_{Pl}^2/t^2$. The non-compensated part of the energy-momentum tensor $\delta T_{\mu\nu} = T_{\mu\nu}^{(vac)} - T_{\mu\nu}^{(\phi)}$ is not necessarily of the vacuum-like form but may have an arbitrary, possibly an exotic, relation between the pressure δp and the energy $\delta \rho$ densities. Under a simplifying assumption that $\delta T_{\mu\nu}$ have the vacuum form, that is $\delta T_{\mu\nu} \sim g_{\mu\nu}$ the idea of the time varying vacuum energy density was lately explored in several papers[16] for construction of realistic cosmologies with time dependent cosmological "constant". Further development along these lines is inhibited by a lack of a consistent Lagrangian model without fine-tuning.

At first glance a scalar field is the only candidate for the adjustment mechanism because its spatially constant classical condensate does not destroy the observed homogeneity and isotropy of the universe. However this is not necessarily the case. For example the time component of a vector field can play this role without violating isotropy and homogeneity[8, 17]. In ref.[8] a gauge vector field with the usual kinetic term $F_{\mu\nu}^2$ was considered. Such field is stable in the De Sitter background and to induce an instability the coupling to the curvature which breaks gauge symmetry was introduced, $\xi RU(A_{\mu}^2)$. The model contains too much arbitrariness, connected with the choice of the potential $U(A^2)$, and gives rise to a time dependent gravitational constant though the dependence can be much milder than in the scalar case, e.g. G_N may logarithmically depend on time.

A more interesting model is based on the gauge non-invariant Lagrangian of the

form[17]:

$$\mathcal{L}_0 = \eta_0 A_{\alpha;\beta} A^{\alpha;\beta} \tag{2}$$

without any potential terms. The classical equation of motion for the time component A_t in this case has indeed an unstable solution and with the proper sign of the constant η_0 the energy-momentum tensor corresponding to this solution compensates the vacuum one. The non-compensated terms die down as $1/t^2$. Unfortunately the cosmology based on this model is not realistic because the scale factor rises too fast, $a(t) \sim t$. We will consider this model in some detail in the next section.

The set of possible compensating fields which do not break the isotropy and homogeneity is not exhausted by a scalar ϕ and vector A_{α} . There can be higher rank tensor fields like e.g. time components of symmetric tensor $S_{\alpha\beta}$ (S_{tt}) or even $S_{\alpha\beta\gamma}$ (S_{ttt}), and isotropic space components like $S_{ij} \sim \delta_{ij}$ or space components of antisymmetric tensor A_{ijk} . With the simplest Lagrangian analogous to (2) these fields (which we denote generically as $V_{\alpha\beta...}$) satisfy the equation of motion

$$D^2 V_{\alpha\beta\dots} = 0 \tag{3}$$

As we see in what follows these fields are also unstable and can change the De Sitter expansion to the power law one, and with a particular choice of the Lagrangian, one can get H = 1/2t or (with a slightly different Lagrangian) H = 2/3t which are already close to realistic cosmologies. Higher rank symmetric tensor fields have not yet been considered but antisymmetric ones naturally appear in high dimensional supergravity or superstring models. These fields however are supposed to be gauge fields but in this case they are stable, so we reject gauge invariance from the very beginning.

It may be not an innocent assumption to introduce massless non-gauge fields and moreover with non-positive definite energy density. There may be even some more serious restrictions on the theory. For example in the case of the vector field A_{α} one has to impose the condition of vanishing of spatial components of the field,

 $A_i = 0$, otherwise these components, which are also unstable would destroy the anisotropy of the universe. This is in a drastic contrast to the normal additional condition $V^{\alpha}_{;\alpha} = 0$ which kills the scalar component. It may create a lot of problems for quantization of such a theory. However our aim here is more modest, it is just to find possible classical solutions of equations of motion which follow from relatively simple Lagrangians and which would kill the cosmological constant. With higher rank tensor fields this problem seem to be easily solved but it may be very difficult (if possible) to make really realistic cosmology. The solution discussed here presents at least a counter example to the "no-go" theorem for the adjustment mechanism, proposed by S. Weinberg in his review[2].

2 Vector field.

Here and in what follows we assume that the background metric is the spatially flat Robertson-Walker one:

$$ds^2 = dt^2 - a^2(t)d\vec{r}^2 (4)$$

The equations of motion (3) for the vector field A_{μ} in this metric have the form:

$$\left(\partial_t^2 - \frac{1}{a^2}\partial_j^2 + 3H\partial_t - 3H^2\right)A_t + \frac{2H}{a^2}\partial_j A_j = 0,\tag{5}$$

$$(\partial_t^2 - \frac{1}{a^2} \partial_j^2 + H \partial_t - \dot{H} - 3H^2) A_j + 2H \partial_j A_t = 0$$
 (6)

where $H = \dot{a}/a$ is the Hubble parameter.

The energy-momentum tensor of this field is easily calculated from the Lagrangian (2) and is equal to:

$$\eta_0^{-1} T_{\mu\nu}(A_\alpha) = -\frac{1}{2} g_{\mu\nu} A_{\alpha;\beta} A^{\alpha;\beta} + A_{\mu;\alpha} A^{;\alpha}_{\nu} + A_{\alpha;\mu} A^{\alpha}_{;\nu} - \frac{1}{2} (A_{\mu;\alpha} A_{\nu} + A_{\nu;\alpha} A_{\mu} + A_{\alpha;\mu} A_{\nu} + A_{\alpha;\nu} A_{\mu} - A_{\mu;\nu} A_{\alpha} - A_{\nu;\mu} A_{\alpha})^{;\alpha}$$
(7)

The Hubble parameter which enters equation (5) is determined by the expression:

$$3H^2m_1^2 = \rho_{tot} = \rho_{vac} + T_{tt} (8)$$

where $m_1^2 = m_{Pl}^2 / 8\pi$.

We will consider a special homogeneous solution: $A_j = 0$ and $A_t = A(t)$. We assume that initially the magnitude of A_t is small and the expansion of the universe is dominated by the vacuum energy, $H_v = \sqrt{8\pi\rho_{vac}/3m_{Pl}^2}$. In this regime A_t exponentially rises, $A_t(t) \sim \exp(0.79Ht)$ and soon its contribution into the energy density becomes non-negligible. If $\eta_0 = -1$ is chosen so that the vacuum energy density and the energy density of the field A_t has opposite signs, the contribution of A_t would diminish H and both the expansion rate and the rate of increase of A_t would slow down. One can check that asymptotically $A_t \sim t$ and H = 1/t. Expanding the solution in powers of 1/t and assuming that $\rho_{vac} > 0$ and $\eta_0 = -1 < 0$ we find:

$$A_t = t\sqrt{\rho_{vac}/2} \left(1 + \frac{c_1}{t} + \frac{c_2}{t^2} \right)$$
 (9)

$$H = \frac{1}{t} \left(1 - \frac{c_1}{t} + \frac{c_1^2 - 4c_2/3}{t^2} \right) \tag{10}$$

where $c_2 = 3m_{Pl}^2/8\pi\rho_{vac}$ and c_1 is determined by initial conditions. The energy and pressure density of this solution are respectively

$$\rho(A_t) = \frac{1}{2}\dot{A}_t^2 + \frac{3}{2}H^2A_t^2 \to \rho_{vac}(-1 + c_2/t^2)$$
(11)

and

$$p(A_t) = \rho_{vac}(1 - c_2/3t^2) \tag{12}$$

From eq.(11) we obtain the following expression for the Hubble parameter:

$$H^{2} = \frac{\rho_{vac} + \eta_{0} \dot{A}_{t}^{2}/2 + \rho_{matter}}{3(m_{1}^{2} - \eta_{0} A_{t}^{2}/2)}$$
(13)

The energy density of normal matter, ρ_{matter} , is added here for generality. Since $\rho_{matter} \sim 1/a^4$ for relativistic matter, ρ_r , and $\sim 1/a^3$ for non-relativistic matter, ρ_{nr} ,

the contribution of the usual matter into total cosmological energy density quickly dies down, $\rho_r \sim 1/t^4$ and $\rho_{nr} \sim 1/t^3$, and becomes negligible. Thus the result H=1/t does not depend on the matter content and follows from the asymptotic rise of the field, $A_t \sim t$. The total cosmological energy density in this model is dominated by the remnant of $(\rho_{vac} - \rho_A) \sim 1/t^2$. This cosmology is not realistic and this is because the expansion rate, $a(t) \sim t$ is too fast. One can try to construct a model with a slower expansion rate using the freedom of adding new derivative terms into the Lagrangian:

$$\mathcal{L}_1 = \eta_1 A_{\mu;\nu} A^{\nu;\mu} \tag{14}$$

$$\mathcal{L}_2 = \eta_2 (A^{\mu}_{;\mu})^2 \tag{15}$$

However the first one gives exactly the same equation of motion for A_t as the Lagrangian \mathcal{L}_0 and the contribution from \mathcal{L}_2 into the equation of motion is just $\eta_2 A^{\alpha}_{;\alpha;\mu}$. It does not change the asymptotic behavior obtained above. So for a more realistic cosmologies one has to address to higher rank fields. We will do that in the next section.

Let us consider now the contribution of the space components A_j into the energy density. It follows from eq. (6) that in the cosmological background with H = 1/t the space components A_j increase as $t^{\sqrt{2}}$ i.e. even faster than A_t , but the energy density of these components remain small in comparison with $\rho(A_t) \approx const$ (11):

$$\rho(A_j) = \frac{1}{a^2} \left(-\frac{1}{2} \dot{A}_j^2 + H \dot{A}_j A_j - H^2 A_j^2 \right) \sim t^{2\sqrt{2}-4} = t^{-1.17}$$
 (16)

However since $\rho(A_t)$ is canceled with ρ_{vac} up to terms of the order $1/t^2$ the contribution of $\rho(A_j)$ becomes dominant. Moreover the energy-momentum tensor of A_j contains undesirable non-isotropic terms proportional to A_iA_j or to \dot{A}_iA_j . These terms can be suppressed if one adds the Lagrangian \mathcal{L}_1 (14) with the proper choice of parameter η_1 . One can check that in this case the space components rise as $A_j \sim t^{\sqrt{2(1+\eta_1/\eta_0)}}$. So for $-1 < \eta_1/\eta_0 < -1/2$ the contribution of A_j into cosmological energy density

would be small. Though the model of this Section is not realistic the tricks used here may be useful for more realistic models considered in the following section.

One more comment about the cosmological solutions with A_t may be of interest. Let us assume now that η_0 is positive, $\eta_0 = 1$. Corresponding cosmological model in this case possesses a rather peculiar singularity. The equation of motion (5) does not change and the field A_t remains unstable in the Robertson-Walker background but the behavior of the solution becomes quite different. One can see from eq.(13) that the Hubble parameter H has a singularity during expansion stage at a finite value of the field amplitude and at a finite time. The solution near the singularity has the form:

$$H = \frac{h_1}{(t_0 - t)^{2/3}},\tag{17}$$

$$A_t(t) = \sqrt{2}m_1 \left[1 + c_1 \left(t_0 - t \right)^{2/3} \right]$$
 (18)

where $m_1 = m_{Pl}/\sqrt{8\pi}$ and c_1 and h_1 are constant. The energy density of the field A_t at the singular point tends to infinity as $(t_0 - t)^{-2/3}$ while the scale factor tends to constant value according to the expression $a(t) \sim \exp[-3h_1(t_0 - t)^{1/3}]$. Since this is not related to the problem of the cosmological constant we will not go into further details and postpone the discussion of the solution with positive η_0 for the future.

3 Higher rank symmetric fields.

Essential features of cosmologies with higher rank symmetric tensor fields are the same as discussed in the previous section but some details may be different and in particular the expansion rate. Equation of motion (3) for the space-point independent components of the second rank symmetric tensor $S_{\alpha\beta}$ in the flat RW background (4) has the form:

$$(\partial_t^2 + 3H\partial_t - 6H^2)S_{tt} - 2H^2S_{jj} = 0 (19)$$

$$(\partial_t^2 + 3H\partial_t - 6H^2)s_{tj} = 0 (20)$$

$$(\partial_t^2 + 3H\partial_t - 2H^2)s_{ij} - 2H^2\delta_{ij}S_{tt} = 0 (21)$$

where $s_{ij} = S_{ij}/a(t)$ and $s_{ij} = S_{ij}/a^2(t)$.

For $\eta_0 = -1$ there exists a particularly interesting homogeneous solution of these equations which at large t behaves as $S_{tt} = Ct$, $s_{ij} = \delta_{ij}Ct/3$, and $s_{tj} = 0$. The condition of vanishing of s_{tj} is not stable and we will return to that below. There may be nonvanishing components s_{ij} which are not proportional to the isotropic tensor δ_{ij} but they rise with time slower than t. The energy density corresponding to this solution

$$\rho = \eta_0 \left[\frac{1}{2} (\dot{S}_{tt}^2 + \dot{s}_{ij}^2) + H^2 (3S_{tt}^2 + s_{ij}^2 + 2S_{tt}s_{jj}) \right]$$
 (22)

exactly compensates the vacuum energy density, as above in the case of vector field, but the expansion rate at large t is different:

$$H = \frac{3}{8t} \tag{23}$$

In this model $a \sim t^{3/8}$ and the energy density of usual matter decreases rather slowly, $\rho_r \sim t^{-3/2}$ and $\rho_{nr} \sim t^{-9/8}$. Corresponding values of the parameter $\Omega = \rho_{matter}/\rho_c$ would be much larger than 1. Though the energy density of the usual matter may be the dominant one, the Hubble parameter, as above, does not depend on it. Using expression (22) we find similar to (13):

$$H^{2} = \frac{\rho_{vac} + \eta_{0}(\dot{S}_{tt}^{2} + \dot{s}_{ij}^{2})/2 + \rho_{matter}}{3m_{1}^{2} - \eta_{0}(3S_{tt}^{2} + s_{ij}^{2} + 2S_{tt}s_{jj})}$$
(24)

One can easily check that the asymptotic solution of the equation of motion $S_{tt} \sim t$ and $s_{ij} \sim t$ gives the result (23) independently of the matter content and its properties, if only ρ_{matter} decreases in the course of expansion. This is in a drastic contrast to the standard cosmology, when the expansion rate is determined by the usual matter, so for an agreement with observations a particular fine-tuning is necessary even if one manages to obtain a normal expansion rate. To achieve the latter we can use the

freedom in the choice of the Lagrangian of the tensor field similar to expressions (14) and (15):

$$\Delta \mathcal{L} = \eta_1 S_{\alpha\beta;\gamma} S^{\alpha\gamma;\beta} + \eta_2 S_{\beta;\alpha}^{\alpha} S_{;\gamma}^{\gamma\beta} + \eta_3 S_{\alpha;\beta}^{\alpha} S_{\gamma}^{\gamma;\beta}$$
 (25)

The corresponding equations of motion can be written as:

$$(\partial_t^2 + 3H\partial_t - 6H^2)S_{tt} - 2H^2s_{jj} +$$

$$C_1[(\partial_t^2 + 3H\partial_t - 3H^2)S_{tt} + (H\partial_t - H^2)s_{jj}] +$$

$$C_2\partial_t[(\partial_t + 3H)S_{tt} + Hs_{jj}] +$$

$$C_3(\partial_t^2 + 3H\partial_t)(S_{tt} - s_{jj}) = 0,$$
(26)

$$(\partial_t^2 + 3H\partial_t - 6H^2)s_{tj} + (C_1/2)(\partial_t^2 + 3H\partial_t - 2\dot{H} - 12H^2)s_{tj} + (C_2/2)[\partial_t(\partial_t + 4H)s_{tj} - H(\partial_t + 4H)s_{tj}] = 0,$$
(27)

$$(\partial_t^2 + 3H\partial_t - 2H^2)s_{ij} - 2H^2\delta_{ij}S_{tt} - C_1[(\dot{H} + 4H^2)s_{ij} + \delta_{ij}(H\partial_t + \dot{H} + 4H^2)S_{tt}] - C_2\delta_{ij}[H^2s_{ll} + (H\partial_t + 3H^2)S_{tt}] - C_3(\partial_t^2 + 3H\partial_t)(S_{tt} - s_{jj}) = 0,$$
(28)

where $C_j = \eta_j/\eta_0$.

As before these equations have unstable solutions $S_{tt} \sim t$ and $s_{ij} \sim \delta_{ij}t$ with $H \sim 1/t$; this solution annihilates the vacuum energy. Varying the parameters η_j we may obtain different expansion regimes and in particular the relativistic expansion H = 1/2t or the non-relativistic one, H = 2/3t. An interesting choice is $C_1 + C_2 = -1$ because it ensures $s_{tj} = 0$. The Hubble parameter corresponding to this choice is given by

$$Ht = \frac{3(C_1+1) + C_3(15+16C_1)}{3(C_1+1) + C_3(20+16C_1)}$$
(29)

Taking e.g. $C_1 = 0$ and $C_3 = -3/10$ we get H = 1/2t as in the radiation dominated universe. So the cosmology does not look as unrealistic as the one in the preceding section, though the reason for the choice of the particular values of C_i is absolutely unknown. In this cosmological model the energy density of relativistic matter and the non-compensated part of the energy density of the $S_{\alpha\beta}$, $\delta\rho = \rho_{vac} - \rho(S_{\alpha\beta})$ both decrease with time as $1/t^2$ and if initially (at $t=t_{Pl}=1/m_{Pl})$ $\rho_r\sim m_{Pl}^4$ and if ρ_{vac} has similar magnitude, then the contribution of relativistic matter into cosmological parameter Ω : $\Omega_r = \rho_r/(3H^2m_{Pl}^2/8\pi)$ would be always around unity. Since it is rather natural to assume that all initial values were close to the Planck ones the equality of ρ_r and $\delta\rho$ does not imply too much of fine-tuning. However the successful results of the theory of primordial nucleosynthesis demands a rather restrictive relation between the rate of the expansion and the energy density of relativistic matter. In this model it is a free parameter which should be rather precisely chosen. This issue is related to the problem of matter creation in the early universe and will be addressed elsewhere. (For a recent analysis of the nucleosynthesis bounds on cosmologies with varying cosmological "constant" see ref.[18].)

Another and a more serious problem of this model is that the Hubble parameter does not depend upon the matter content of the universe, as it has been mentioned above. Correspondingly the expansion would always remain relativistic and this may create some rather evident cosmological problems. It may be interesting to construct a model in which the parameters η_j vary together with the expansion in such a way that the regime changes from the relativistic to nonrelativistic one (for the case considered above it is achieved for $C_3 = -3/5$). However a natural mechanism for realization of such a scenario is not found. Still though the model does not look realistic one may hope that in a more complicated version it will describe our universe.

4 Discussion.

There are two questions which are vitally important for the proposed model. First one is rather philosophical, whether it worthwhile to make such a construction for killing vacuum energy. The assumptions made above about the form of the Lagrangian are not absolutely harmless. Quantum version of this model would definitely meet very strong difficulties like negative probabilities, nonrenormalizability, etc. Quantum corrections may dramatically change the form of the Lagrangian in particular generating nonzero masses and/or some other terms which may depend on the field but not its derivative. If so, the model would not work.

On the other hand there is absolutely no way, known at the present day, how one can get rid of huge vacuum energy which would make our life impossible in such a universe. One can of course invoke the antropic principle but in many cases it is very close to the assumption of a supernatural creation of the world. If this is the case cosmologists would be jobless. This model at least propose a mechanism of automatic cancellation of ρ_{vac} with time independent gravitational constant, which was a drawback of previous attempts. It gives reasonable theoretical frameworks (at classical level) for cosmology with asymptotically vanishing vacuum energy based on Lagrangian approach.

The second question is if it is possible at all to construct a realistic cosmology based on this approach. There is a subquestion if the realistic model would be natural or demands a strong fine-tuning like the fine-tuning of the vacuum energy in the traditional cosmology. The answer to the last part of the question is semi-negative. At the moment no way is seen how to make a natural model but the necessary fine-tuning is possibly not as strong as 10^{100} . On the other hand one may impose ad hoc the necessary values of the parameters and to use this model as a toy one to study cosmology with automatic cancellation of Λ . In particular the model considered here

gives the non-compensated amount of vacuum energy of the order of $\rho_c \sim m_{Pl}^2/t^2$. As it was argued in ref.[6, 8] this is a generic phenomenon. One more problem which may be inherent for this kind of cosmological model is that inflation is very difficult or even impossible to realize. The vacuum (or vacuum-like) energy is destroyed so fast that inflation stops before it starts. To avoid it one has to suppress somehow the instability of the fields when their amplitude is small. More work in this and many other directions is necessary.

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